בס"ד

First Greater Boston Math Olympiad
6th Grade

Your name: _____

Try to solve as many problems as you can, in any order you choose. There are six problems, and a correct solution of each of them wins you the number of points shown in parentheses. Show your work. If your answer is wrong but your method is correct, you will get partial credit. If necessary, use back sides of pages or attach additional sheets, putting your name on them.

Good luck!

PLEASE DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

1. (6 points) Solve: BOOK + BOOK + BOOK + BOOK + BOOK + BOOK = TEST
(BOOK and TEST are 4-digit numbers, and different letters stand for different digits).

Answer:

You can use the space below for the scratch work.

2. (6 points) The number A2...2B has 2004 digits (all digits standing between A and B are 2). This number is divisible by 72. Find the digits A and B. (**Hint**: use the fact that $72 = 8 \times 9$. Find B first and then A.)

Answer:

Explanation:

3. A number N is divisible by 18 and has exactly 10 divisors (including 1 and N).

(**a**) (7 points) Find such N.

Answer:

Explanation:

(**b**) (8 points) Is it unique?

Answer:

Explanation:

4. Two people play a game. They put 3 piles of matches on the table:
the first one contains 2 matches, the second one 3 matches, and the third one 4 matches.
Then they take turns making moves. In a move, a player may take any nonzero number
of matches FROM ONE PILE. The player who takes the last match from the table
loses the game.

(**a**) (5 points) The player who makes the first move can win the game.
What is the winning first move?

Answer:

(**b**) (6 points) How can he win? (Describe his strategy.)

Answer:

5. (**a**) (4 points) How many times in a day (= 24 hours) the hour and the minute hand of a clock form the right angle with each other?

Answer:

Explanation:

(**b**) (8 points) How many times in a day the seconds hand of a clock falls on the line bisecting the angle between the hour and the minute hands?

Answer:

Explanation:

6. (**a**) (3 points) Put 5 points on the plane so that each 3 of them are vertices of an isosceles triangle (i.e., a triangle with two equal sides), and no three points lie on the same line.

(**b**) (7 points) Do the same with 6 points.

Answers:

You can use the space below for the scratch work.